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FUZZY OPTIMIZATION INVENTORY MODEL USING KUHN-TUCKER TECHNIQUE

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ABSTRACT
This paper preambles two fuzzy production inventory models under fuzzy parameters for crisp production quantity or for fuzzy production quantity. The fuzzy total production inventory costs of these models along with the fuzzy arithmetical operations of function principle are proposed. The main purpose is to find optimal solutions of these models by using ranking function method for defuzzifying the fuzzy total production inventory cost and by using Kuhn-Tucker method for solving inequality constraint problem. In addition, when the fuzzy parameters [Fuzzy inventory cost, fuzzy demand, fuzzy setup cost, fuzzy demand rate, and fuzzy production rate] are all crisp real numbers. The optimal solutions of our proposed models can be specified to meet classical production inventory models.

KEYWORDS: Fuzzy production inventory, production inventory, Function principle, Ranking function, Kuhn-Tucker method.

I. INTRODUCTION
The fuzzy set concept has been used to treat the classical inventory model recently. Park (1987) introduced fuzzy inventory cost in economic order quantity model. Chang (1999) discussed how to obtain the economic production quantity, when the quantity of demand is uncertain. Chen and Hseih (2000) established a fuzzy economic production model to treat the inventory problem with all the parameters and variables being fuzzy numbers. Hsieh (2002), Lee and Yao (1998) and Lin and Yao (2000) also wrote same papers about the fuzzy production model.

In view of production management, the cost should include explicit and implicit, and the calculation should be easy. We have the implicit cost in this model and use function principle to calculate the fuzzy total production inventory cost and use the ranking function method to defuzzify the FTPIC. At first, the methodology is introduced. Then two models are discussed and a numerical example is shown and finally conclusion is derived.

II. METHODOLOGY
In this paper, we use the function principle and ranking function method to find the optimal economic production quantity with a fuzzy inventory model. When the quantities are fuzzy numbers, we need to use the Kuhn-Tucker conditions to solve the model. Therefore, we introduce these three methodologies as follows.

(i) Ranking Function Method
We define a ranking function $R: f(R) \to R$ which maps each fuzzy numbers to the real line: f(R) represents the set of all Trapezoidal fuzzy numbers. If R be any linear ranking function, then

$$R(\tilde{A}) = \left(\frac{a_1 + a_2 + a_3 + a_4}{4} \right)$$

(ii) The Kuhn-Tucker Conditions
The development of the Kuhn-Tucker conditions is based on the Lagrangean method.

Suppose that the problem is given by,
Minimize Y=f(x)
Subject to $g_i(x) \geq 0, i = 1, 2.....m$

The non negativity constraints may be converted into equations by using non negative surplus variables. Let,
$\lambda = (\lambda_1, \lambda_2, ..., \lambda_m), g(x) = (g_1(x), g_2(x), ...g_m(x)).....and$
$s^2 = (s_1^2, s_2^2, ...s_m^2).$

The Kuhn-Tucker conditions need x and λ to be a stationary point of the minimization problem, which can be summarized as follows:

$$\left\{\begin{array}{l} \lambda_1 \leq 0 \\ \nabla f(x) - \lambda \nabla g(x) = 0, \\ \lambda_i g_i(x) = 0, i = 1, 2, ..., m \\ g_i(x) \geq 0, i = 1, 2, ...m \end{array}\right\}$$

Fuzzy arithmetical operations under function principle

We describe some fuzzy arithmetical operations under function principle as follows.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two Trapezoidal fuzzy numbers. Then

1. The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$ where a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 are any real numbers.
2. The multiplications of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4)$ where T= $\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, c_1 =min T, c_2 =min T_1, c_3 =max T, c_4 =max T_1. If $a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4$ are all non zero positive real numbers, then $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
3. $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$ where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are any real numbers
4. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2, b_3, b_4 are all positive real numbers. If $a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4$ are all non zero positive real numbers, Then the division of $\tilde{A}$ and $\tilde{B}$ is $\alpha \in R \tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
5. Let $\alpha \in R$ then,
   - $\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$
   - $\alpha < 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

For crisp production quantity

$$Q_p^* = \sqrt{\frac{2TD}{A\left(1 - \left(\frac{R}{P}\right)\right)}}$$

Q_p^* **=4256.66**

Fuzzy Production Inventory Model using ranking method

Notations:

$\tilde{D}$ - Fuzzy Yearly demand
$\tilde{A}$ - Fuzzy Inventory cost
$\tilde{T}$ - Fuzzy setup cost
$\tilde{P}$ - Fuzzy daily production rate
$\tilde{R}$ - Fuzzy daily demand rate
$\tilde{Q}_p$ - Fuzzy production quantity
Q_p - Crisp production quantity

Now, we introduce fuzzy production inventory model with fuzzy parameters for crisp production quantity Q_p as follows

In this model, the annual inventory cost FIC_1 and fuzzy annual setup cost FSC_1 is

$$FIC_1 = \tilde{A} \otimes \frac{Q_p}{2} \otimes \left[1 \Theta \left(\tilde{R} \oslash \tilde{P} \right) \right] \text{ and } FSC_1 = \tilde{T} \otimes \left(\tilde{D} \oslash Q_p \right)$$

Then the total production inventory cost is

$$\tilde{C}_1 = FIC_1 + FSC_1 = \tilde{T} \otimes \left(\tilde{D} \oslash Q_p \right) \oplus \tilde{A} \otimes \frac{Q_p}{2} \otimes \left[1 \Theta \left(\tilde{R} \oslash \tilde{P} \right) \right],$$

where $\oslash, \otimes, \oplus, \Theta$ are the fuzzy arithmetical operations under function principle.

Here we suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{D} = (d_1, d_2, d_3, d_4,)$, $\tilde{T} = (t_1, t_2, t_3, t_4,)$, $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{R} = (r_1, r_2, r_3, r_4,)$ are non-negative trapezoidal fuzzy numbers. Then we solve the optimal production quantity of formula as the following steps.

First, we get the fuzzy total production inventory cost $\tilde{C}_1$ by formula as,

$$\tilde{C}_1 = \begin{bmatrix} \frac{t_1 d_1}{Q_p} + \frac{a_1 Qp}{2}\left(1 - \frac{r_4}{p_1}\right), \frac{t_2 d_2}{Q_p} + \frac{a_2 Q_p}{2}\left(1 - \frac{r_3}{p_2}\right), \\ \frac{t_3 d_3}{Q_p} + \frac{a_3 Q_p}{2}\left(1 - \frac{r_2}{p_3}\right), \frac{t_4 d_4}{Q_p} + \frac{a_4 Q_p}{2}\left(1 - \frac{r_4}{p_1}\right) \end{bmatrix}$$

Second, we defuzzifying the fuzzy total production inventory cost using ranking function method. The result is,

$$P(\tilde{C}_1) = \frac{1}{4} \begin{bmatrix} \frac{t_1 d_1}{Q_p} + \frac{a_1 Qp}{2}\left(1 - \frac{r_4}{p_1}\right) + \frac{t_2 d_2}{Q_p} + \frac{a_2 Q_p}{2}\left(1 - \frac{r_3}{p_2}\right) + \\ \frac{t_3 d_3}{Q_p} + \frac{a_3 Q_p}{2}\left(1 - \frac{r_2}{p_3}\right) + \frac{t_4 d_4}{Q_p} + \frac{a_4 Q_p}{2}\left(1 - \frac{r_4}{p_1}\right) \end{bmatrix}$$

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$$=\frac{1}{4}\left[\frac{1}{Q_p}(t_1d_1+t_2d_2+t_3d_3+t_4d_4)+\right]+Q_p\left[\frac{a_1}{2}\left(1-\frac{r_4}{p_1}\right)+\frac{a_2}{2}\left(1-\frac{r_3}{p_2}\right)+\frac{a_3}{2}\left(1-\frac{r_2}{p_3}\right)+\frac{a_4}{2}\left(1-\frac{r_1}{p_4}\right)\right]$$

$$\frac{\partial\left[P(\tilde{C}_1)\right]}{\partial Q_p}=\frac{1}{4}\left[\frac{-1}{Q_p{}^2}(t_1d_1+t_2d_2+t_3d_3+t_4d_4)+\frac{a_1}{2}\left(1-\frac{r_4}{p_1}\right)+\frac{a_2}{2}\left(1-\frac{r_3}{p_2}\right)+\frac{a_3}{2}\left(1-\frac{r_2}{p_3}\right)+\frac{a_4}{2}\left(1-\frac{r_1}{p_4}\right)\right]$$

$$\frac{\partial\left[P(\tilde{C}_1)\right]}{\partial Q_p}=0\Rightarrow\frac{a_1}{2}\left(1-\frac{r_4}{p_1}\right)+\frac{a_2}{2}\left(1-\frac{r_3}{p_2}\right)+\frac{a_3}{2}\left(1-\frac{r_2}{p_3}\right)+\frac{a_4}{2}\left(1-\frac{r_1}{p_4}\right)=\frac{1}{Q_p{}^2}(t_1d_1+t_2d_2+t_3d_3+t_4d_4)$$

$$\frac{Q_p^2}{2}\left[a_1\left(1-\frac{r_4}{p_1}\right)+a_2\left(1-\frac{r_3}{p_2}\right)+a_3\left(1-\frac{r_2}{p_3}\right)+a_4\left(1-\frac{r_1}{p_4}\right)\right]=(t_1d_1+t_2d_2+t_3d_3+t_4d_4)$$

$$Q_p^2=\frac{2(t_1d_1+t_2d_2+t_3d_3+t_4d_4)}{\left[a_1\left(1-\frac{r_4}{p_1}\right)+a_2\left(1-\frac{r_3}{p_2}\right)+a_3\left(1-\frac{r_2}{p_3}\right)+a_4\left(1-\frac{r_1}{p_4}\right)\right]}$$

$$Q_p^*=\sqrt{\frac{2(t_1d_1+t_2d_2+t_3d_3+t_4d_4)}{\left[a_1\left(1-\frac{r_4}{p_1}\right)+a_2\left(1-\frac{r_3}{p_2}\right)+a_3\left(1-\frac{r_2}{p_3}\right)+a_4\left(1-\frac{r_1}{p_4}\right)\right]}}$$

The fuzzy production inventory model for fuzzy production quantity

In this section we introduce the fuzzy production inventory models by changing the crisp production quantity into fuzzy production quantity.

Suppose fuzzy production quantity $\tilde{Q}_p$ be a trapezoidal fuzzy number $\tilde{Q}_p=(qp_1,qp_2,qp_3,qp_4)$ with $0<qp_1\le qp_2\le qp_3\le qp_4$. The fuzzy annual inventory cost FIC_2 and the fuzzy setup cost FSC_2 of this model are $FIC_2=\tilde{A}\otimes(\tilde{Q}_p\oslash 2)\otimes\left[1\ominus\tilde{R}\oslash\tilde{P}\right] and FSC_2=\tilde{T}\otimes(\tilde{D}\oslash\tilde{Q}_p)$.

(iii) For Fuzzy Production Quantity:

Notation:

$\tilde{Q}_p$ - Fuzzy production quantity
$\tilde{D}$ - Fuzzy Yearly demand
$\tilde{A}$ - Fuzzy Inventory cost
$\tilde{T}$ - Fuzzy setup cost
$\tilde{P}$ - Fuzzy daily production rate
$\tilde{R}$ - Fuzzy daily demand rate

Fuzzy total production inventory cost $\tilde{C}_2=FSC_2+FIC_2$

$$\tilde{C}_2=\left[\frac{t_1d_1}{qp_4}+\frac{a_1qp_1}{2}\left(1-\frac{r_4}{p_1}\right),\frac{t_2d_2}{qp_3}+\frac{a_2qp_2}{2}\left(1-\frac{r_3}{p_2}\right),\frac{t_3d_3}{qp_2}+\frac{a_3qp_3}{2}\left(1-\frac{r_2}{p_3}\right),\frac{t_4d_4}{qp_1}+\frac{a_4qp_4}{2}\left(1-\frac{r_1}{p_4}\right)\right]$$

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By ranking function, we obtain

$$P\left(\tilde{C}_2\right)=\frac{1}{4}\left[\frac{t_1d_1}{qp_4}+\frac{a_1qp_1}{2}\left(1-\frac{r_4}{p_1}\right)+\frac{t_2d_2}{qp_3}+\frac{a_2qp_2}{2}\left(1-\frac{r_3}{p_2}\right)+\frac{t_3d_3}{qp_2}+\frac{a_3qp_3}{2}\left(1-\frac{r_2}{p_3}\right)+\frac{t_4d_4}{qp_1}+\frac{a_4qp_4}{2}\left(1-\frac{r_1}{p_4}\right)\right]$$

The Kuhn-Tucker condition is used to find the solution of qp_1, qp_2, qp_3 and qp_4 to minimize $P\left(\tilde{C}_2\right)$, subject to $qp_2-qp_1\ge 0$, $qp_3-qp_2\ge 0$, $qp_4-qp_3\ge 0$ and $qp_1>0$.

The Kuhn Tucker conditions are,

$$\lambda\le 0$$

$$\nabla fP\left(\tilde{C}_2\right)-\lambda\nabla g\left(Q\right)=0$$

$$\lambda_i g_i\left(Q\right)=0$$

$$g_i\left(Q\right)\ge 0$$

These conditions can be simplified to the following $\lambda_1,\lambda_2,\lambda_3,\lambda_4\le 0$

$$\left.\begin{aligned}
&-\frac{t_4d_4}{qp_1}+\frac{a_4qp_4}{2}\left(1-\frac{r_1}{p_4}\right)+\lambda_1-\lambda_4=0\\
&-\frac{t_3d_3}{qp_2}+\frac{a_3qp_3}{2}\left(1-\frac{r_2}{p_3}\right)-\lambda_1-\lambda_2=0\\
&-\frac{t_2d_2}{qp_3}+\frac{a_2qp_2}{2}\left(1-\frac{r_3}{p_2}\right)-\lambda_2+\lambda_3=0\\
&-\frac{t_1d_1}{qp_4}+\frac{a_1qp_1}{2}\left(1-\frac{r_4}{p_1}\right)-\lambda_3=0
\end{aligned}\right\}\text{L L L L L}\ (1)$$

If $qp_1>0 and \lambda_4 qp_1=0, \lambda_4=0. If\ \lambda_1=\lambda_2=\lambda_3=\lambda_4$. then $qp_4<qp_3<qp_2<qp_1$. It does not satisfy the constraints $0<qp_1\le qp_2\le qp_3\le qp_4$

$$\therefore\quad qp_2=qp_1, qp_3=qp_2, qp_4=qp_3$$

$$(i.e)qp_1=qp_2=qp_3=qp_4=Q^*$$

From Formula (1), we find the optimal production quantity Q^* by the above equation as,

$$\frac{-1}{Q^*}\left[t_1d_1+t_2d_2+t_3d_3+t_4d_4\right]+\frac{Q^*}{2}\left[a_1\left(1-\frac{r_4}{P_1}\right)+a_2\left(1-\frac{r_3}{P_2}\right)+a_3\left(1-\frac{r_2}{P_3}\right)+a_4\left(1-\frac{r_1}{P_4}\right)\right]=0$$

$$\Rightarrow\frac{Q^*}{2}\left[a_1\left(1-\frac{r_4}{P_1}\right)+a_2\left(1-\frac{r_3}{P_2}\right)+a_3\left(1-\frac{r_2}{P_3}\right)+a_4\left(1-\frac{r_1}{P_4}\right)\right]=\frac{1}{Q^*}\left[t_1d_1+t_2d_2+t_3d_3+t_4d_4\right]$$

$$\Rightarrow Q^{*2}=\frac{2\left(t_1d_1+t_2d_2+t_3d_3+t_4d_4\right)}{\left[a_1\left(1-\frac{r_4}{P_1}\right)+a_2\left(1-\frac{r_3}{P_2}\right)+a_3\left(1-\frac{r_2}{P_3}\right)+a_4\left(1-\frac{r_1}{P_4}\right)\right]}$$

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$$\Rightarrow Q^* = \sqrt{\frac{2\left(t_1d_1 + t_2d_2 + t_3d_3 + t_4d_4\right)}{\left[a_1\left(1-\frac{r_4}{P_1}\right) + a_2\left(1-\frac{r_3}{P_2}\right) + a_3\left(1-\frac{r_2}{P_3}\right) + a_4\left(1-\frac{r_1}{P_4}\right)\right]}}$$

Numerical Example

Brown manufacturing company produces commercial refrigeration units in batches. The firms estimated demand for the year is greater or less than $10500 units. The setup cost is about $100 and the inventory cost is about $0.5 per unit per year. Once the production process has been setup, greater or less than 90 refrigeration units can be manufactured daily. How many refrigerator units should Brown manufacturing company produce in each batch?

Solution

Here, we use a general rule to transfer the linguistic data "greater or less than X" or "about X", into trapezoidal fuzzy numbers as greater or less than X=(0.95X,1.0X,1.1X,1.15X) and about X= (0.95X,X,X,1.05X).

By the above rule, the fuzzy parameters can be transferred as follows;

Fuzzy yearly demand:

"greater or less than 10500" $\tilde{D} = (d_1, d_2, d_3, d_4)$ = (9500, 10000,10000,10500)

Fuzzy setup cost:

"about $100" $\tilde{T} = (t_1, t_2, t_3, t_4)$ = (95,100,100,105)

Fuzzy inventory cost:

"about $0.5" = $\tilde{a} = (a_1, a_2, a_3, a_4)$ = (0.475,0.5,0.5,0.525)

Fuzzy daily demand rate:

"about 70" $\tilde{R} = (r_1, r_2, r_3, r_4)$ = (67,70,70,73)

Hence the Fuzzy production quantity $\tilde{Q}_p = (qp_1, qp_2, qp_3, qp_4)$ with $0 < qp_1 \le qp_2 \le qp_3 \le qp_4$ is given by replacing the above fuzzy parameter values into the formula. Using this, we find the optimal fuzzy production quantity.

$\tilde{Q}_p$ = (4379.74, 4379.74,4379.74,4379.74)

And the minimization fuzzy total production inventory cost is,

$\tilde{C}_2$ = (320.27, 431.97, 530.80, 639.45)

$$qp^* = \sqrt{\frac{2(t_1d_1 + t_2d_2 + t_3d_3 + t_4d_4)}{a_1\left[1-\left(\frac{r_4}{p_1}\right)\right] + a_2\left[1-\left(\frac{r_3}{p_2}\right)\right] + a_3\left[1-\left(\frac{r_2}{p_3}\right)\right] + a_4\left[1-\left(\frac{r_1}{p_4}\right)\right]}}$$

= 4379.74

$$\tilde{C}_2 = \left[\frac{t_1d_1}{q_p} + \frac{a_1q_p}{2}\left(1-\frac{r_4}{p_1}\right), \frac{t_2d_2}{q_p} + \frac{a_2q_p}{2}\left[1-\frac{r_3}{p_2}\right], \frac{t_3d_3}{q_p} + \frac{a_3q_p}{2}\left[1-\frac{r_2}{p_3}\right], \frac{t_4d_4}{q_p} + \frac{a_4q_p}{2}\left[1-\frac{r_1}{p_4}\right]\right] \tilde{C}_2$$

= (320.27, 431.97, 530.80, 639.45)

III. SENSITIVITY ANALYSIS

S. No	**Demand (D)**	**Fuzzy setup Cost(T)**	**Fuzzy inventory Cost(a)**	**Production rate(P)**	**Daily demand cost (R)**	qp^*	$\breve{c}_2$
1.	10000	(95,100,100,105)	(0.475,0.5,0.5,0.52)	(72,76,84,88)	(57,60,60,63)	4036.6	(331.64,447.79, 548.44,659.39)
2.	10500	(95,100,100,105)	(0.475,0.5,0.5,0.52)	(82,86,94,98)	(67,70,70,73)	4379.74	(320.27,431.97, 530.80,639.45)
3.	11000	(100,105,105,110)	(0.575,0.6,0.6,0.625)	(76,78,82,84)	(66,68,72,74)	4696.04	(248.46,352.14, 525.47,595.5)
4.	12000	(100,105,105,110)	(0.575,0.6,0.6,0.625)	(86,88,92,94)	(76,78,82,84)	4392.00	(254.49,352.69, 511.29,613.44)

IV. CONCLUSION

In the fuzzy environment, it may be possible and reasonable to discuss the fuzzy production inventory models with trapezoidal fuzzy number for crisp production quantity Q_p or for fuzzy production quantity Q_p. In addition, we find that the optimal fuzzy production quantity $\tilde{Q}_p = (q_p^*, q_p^*, q_p^*, q_p^*)$ is the special type of trapezoidal fuzzy number. It can also be considered as crisp real number and the optimal solutions of our proposed models Q_p^* and $\breve{Q}_p^*$ are all coincides .From the sensitivity analysis if we increase the demand and the cost (setup cost, inventory cost, production cost and daily demand cost) then the economic order quantity will be increased ,but the total cost will be decreased

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